

(b) $A \cap C$

Solution Set A consists of First Solar, IBM, Netflix, Procter & Gamble, and Verizon, and set C consists of First Solar, Ford, IBM, and Verizon. Hence the stocks in both A and C are

$$A \cap C = \{\text{First Solar, IBM, Verizon}\}.$$

(c) $A \cup B$

Solution $A \cup B = \{\text{First Solar, IBM, Netflix, Procter & Gamble, and Verizon}\}$

Checkpoint 10

In Example 9, find the given set of stocks.

- (a) $B \cap C$
(b) $B \cup C$

8.1 Exercises

Write true or false for each statement. (See Example 1.)

- $3 \in \{2, 5, 7, 9, 10\}$
- $6 \in \{-2, 6, 9, 5\}$
- $9 \in \{2, 1, 5, 8\}$
- $3 \in \{7, 6, 5, 4\}$
- $\{2, 5, 8, 9\} = \{2, 5, 9, 8\}$
- $\{3, 7, 12, 14\} = \{3, 7, 12, 14, 0\}$
- $\{\text{all whole numbers greater than 7 and less than 10}\} = \{8, 9\}$
- $\{\text{all natural numbers not greater than 3}\} = \{0, 1, 2\}$
- $\{x|x \text{ is an odd integer, } 6 \leq x \leq 18\} = \{7, 9, 11, 15, 17\}$
- $\{x|x \text{ is a vowel}\} = \{a, e, i, o, u\}$
- The elements of a set may be sets themselves, as in $\{1, \{1, 3\}, \{2\}, 4\}$. Explain why the set $\{\emptyset\}$ is not the same set as $\{\emptyset\}$.
- What is set-builder notation? Give an example.

Let $A = \{-3, 0, 3\}$, $B = \{-2, -1, 0, 1, 2\}$, $C = \{-3, -1\}$, $D = \{0\}$, $E = \{-2\}$, and $U = \{-3, -2, -1, 0, 1, 2, 3\}$. Insert $\subset$ or $\not\subset$ to make the given statements true. (See Example 2.)

- $A \subset U$
- $E \subset A$
- $A \subset E$
- $B \subset C$
- $\emptyset \subset A$
- $\{0, 2\} \subset D$
- $D \subset B$
- $A \subset C$

Find the number of subsets of the given set. (See Example 5.)

- $\{A, B, C\}$
- $\{\text{red, yellow, blue, black, white}\}$
- $\{x|x \text{ is an integer strictly between 0 and 8}\}$
- $\{x|x \text{ is a whole number less than 4}\}$

Find the complement of each set. (See Example 6.)

- The set in Exercise 23 if U is the set of all integers.
- The set in Exercise 24 if U is the set of all whole numbers.

27. Describe the intersection and union of sets. How do they differ?

Insert $\cap$ or $\cup$ to make each statement true. (See Examples 7 and 8.)

- $\{5, 7, 9, 19\} _____ \{7, 9, 11, 15\} = \{7, 9\}$
- $\{8, 11, 15\} _____ \{8, 11, 19, 20\} = \{8, 11\}$
- $\{2, 1, 7\} _____ \{1, 5, 9\} = \{1\}$
- $\{6, 12, 14, 16\} _____ \{6, 14, 19\} = \{6, 14\}$
- $\{3, 5, 9, 10\} _____ \emptyset = \emptyset$
- $\{3, 5, 9, 10\} _____ \emptyset = \{3, 5, 9, 10\}$
- $\{1, 2, 4\} _____ \{1, 2, 4\} = \{1, 2, 4\}$
- $\{1, 2, 4\} _____ \{1, 2\} = \{1, 2, 4\}$

36. Is it possible for two nonempty sets to have the same intersection and union? If so, give an example.

Let $U = \{a, b, c, d, e, f, 1, 2, 3, 4, 5, 6\}$, $X = \{a, b, c, 1, 2, 3\}$, $Y = \{b, d, f, 1, 3, 5\}$, and $Z = \{b, d, 2, 3, 5\}$.

List the members of each of the given sets, using set braces. (See Examples 6–8.)

- $X \cap Y$
- $X \cup Y$
- X'
- Y'
- $X' \cap Y'$
- $X' \cap Z$
- $X \cup (Y \cap Z)$
- $Y \cap (X \cup Z)$

Let $U = \{\text{all students in this school}\}$,
 $M = \{\text{all students taking this course}\}$,
 $N = \{\text{all students taking accounting}\}$, and
 $P = \{\text{all students taking philosophy}\}$.

Describe each of the following sets in words.

- M'
- $M \cup N$
- $N \cap P$
- $N' \cap P'$

49. Refer to the sets listed in the directions for Exercises 13–20. Which pairs of sets are disjoint?

50. Refer to the sets listed in the directions for Exercises 37–44. Which pairs of sets are disjoint?

Refer to Example 9 in the text. Describe each of the sets in Exercises 51–54 in words; then list the elements of each set.

51. A'
52. $B \cup C'$
53. $A' \cap B'$
54. $B' \cup C$

Business A stock broker classifies her clients by sex, marital status, and employment status. Let the universal set be the set of all clients, M be the set of male clients, S be the set of single clients, and E be the set of employed clients. Describe the following sets in words.

- $$\begin{array}{ll} 55. M \cap E & 56. M' \cap S \\ 57. M' \cup S' & 58. E' \cup S' \end{array}$$

Food and Drink Sales Sales (in millions of dollars) in the United States at eating and drinking establishments in the years 2014 and 2015 are shown in the following table. (Data from: National Restaurant Association.)

Type of Place or Service	2014	2015
Table-service restaurants (<i>T</i>)	213,498	219,690
Quick-service and fast-casual restaurants (<i>Q</i>)	192,836	201,128
Cafeterias, grill-buffets, and buffets (<i>C</i>)	8,429	8,336
Snack and nonalcoholic beverage bars (<i>S</i>)	31,221	32,845
Bars and taverns (<i>B</i>)	19,943	20,562
Other (<i>O</i>)	8,637	9,076

List the elements of each set.

59. The set of eating and drinking establishments that generated more than \$200,000 million in revenue in both 2014 and 2015.
60. The set of eating and drinking establishments that generated less than \$20,000 million in revenue in 2014 or 2015.
61. The set of eating and drinking establishments that had revenues fall from 2014 to 2015.
62. The set of eating and drinking establishments that had revenues rise from 2014 to 2015 and had less than \$35,000 million in revenue.

Cable Subscribers The top six basic cable television providers in the year 2016 are listed in the following table. Use this information for Exercises 63–72.

Rank	Cable Provider	Subscribers (millions)
1	Direct TV	25.3
2	Comcast	22.5
3	Charter	17.5
4	Dish	13.9
5	Verizon FIOS	4.7
6	Altice	4.6

List the elements of the following sets. (Data from: www.ncta.com.)

63. F , the set of cable providers with more than 5 million subscribers.

64. G , the set of cable providers with between 10 million and 20 million subscribers.
65. H , the set of cable providers with less than 20 million subscribers.
66. I , the set of cable providers with more than 15 million subscribers.
67. $F \cup G$
68. $H \cap F$
69. I'
70. $I' \cap H$
71. J , the set of cable providers with more than 30 million subscribers.
72. $(F \cup H)'$

Farm Products The following table gives the amount (in million metric tons) of several farm products exported from the United States in 2011 and 2016. (Data from: U.S. Department of Agriculture.)

Product	2011	2016
Wheat	28.6	28.2
Corn	39.4	56.5
Rice	3.2	3.6
Barley	.2	.1

List the elements of the following sets.

73. The set of farm products where exports increased from 2011 to 2016
74. The set of farm products that had more than 40 million metric tons of exports in both 2011 and 2016
75. The set of farm products that had more than 50 million metric tons of exports in both 2011 and 2016
76. The set of farm products that had less than 40 million metric tons of exports in either 2011 or 2016
77. The set of farm products that had more than 3.5 million metric tons of exports in either 2011 or 2016
78. The set of farm products that decreased in exports from 2011 to 2016 and had more than 30 million metric tons of exports in 2011

Checkpoint Answers

1. (a) False (b) True (c) True
2. (a) $\{6, 7\}$ (b) $\{-2, -1, 0, 1\}$
3. (a) False (b) True (c) True (d) False
4. $\emptyset, \{w\}, \{x\}, \{y\}, \{z\}, \{w, x\}, \{w, y\}, \{w, z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{w, x, y\}, \{w, x, z\}, \{w, y, z\}, \{x, y, z\}, \{w, x, y, z\}$
5. (a) 16 (b) 128 (c) 2
6. (a) No (b) Yes (c) Yes (d) Yes
7. (a) $\{a, b, c\}$ (b) $\{b, f\}$
8. $\{3\}$
9. $\{a, b, c, e\}$
10. (a) $\{\text{First Solar, Verizon}\}$
(b) $\{\text{First Solar, Ford, IBM, Verizon}\}$

Find the number of people in the given sets.

(a) A

Solution The set A consists of males and females who worked full-time. From the table, we see that there were 70,567 thousand males and 53,194 thousand females, which yields 123,761 thousand people who worked full-time in the year 2016.

(b) $F \cap A$

Solution The set $F \cap A$ consists of all the people who are female *and* who worked full-time. We see that that there were 53,194 thousand females who worked full-time.

(c) $M \cup C$

Solution The set $M \cup C$ consists of people who are male or people who were looking for work. Using the addition rule for counting, we have

$$n(M \cup C) = n(M) + n(C) - n(M \cap C).$$

From the table, we have $n(M) = 70,567 + 10,002 + 4187 = 84,756$ thousand males. Also from the table we have $n(C) = 4187 + 3565 = 7752$ thousand people who are looking for work. We also have $n(M \cap C)$, the number of people who are male and are looking for work, is 4187 thousand. Thus,

$$\begin{aligned} n(M \cup C) &= n(M) + n(C) - n(M \cap C) \\ &= 84,756 + 7752 - 4187 \\ &= 88,321 \text{ thousand people.} \end{aligned}$$

Checkpoint 7

Refer to Example 7 and find the number of people in the following sets.

(a) $A \cup B$

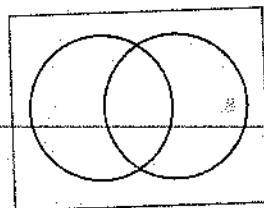
(b) $F \cup B$

(d) $(B \cup C) \cap F$

Solution Begin with the set $B \cup C$, which contains all those who worked part-time or were looking for work. Of this set, take only those who were females, for a total of $17,674 + 3565 = 21,239$ thousand. 

8.2 Exercises

Sketch a Venn diagram like the one shown, and use shading to show each of the given sets. (See Example 1.)



1. $A \cap B'$

2. $A \cup B'$

3. $B' \cup A'$

4. $A' \cap B'$

5. $B' \cup (A \cap B')$

6. $(A \cap B) \cup A'$

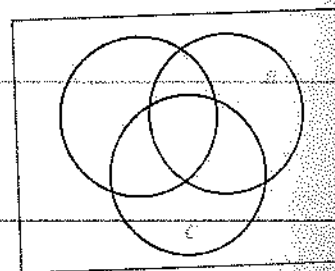
7. U'

8. $\emptyset'$

9. Three sets divide the universal set into at most _____ regions.

10. What does the notation $n(A)$ represent?

Sketch a Venn diagram like the one shown, and use shading to show each of the given sets. (See Example 2.)



11. $(A \cap C') \cup B$

12. $A \cap (B \cup C)$

13. $A' \cap (B \cap C)$

14. $(A' \cap B) \cap C'$

15. $(A \cap B') \cup C$

16. $(A \cap B') \cap C$

Use Venn diagrams to answer the given questions. (See Examples 3, 4, and 5.)

17. **Movies** In 2016, among the top 100 grossing movies in the United States, 15 were rated PG-13 and earned over \$100 million. The number of movies that were rated PG-13 and earned less than \$100 million was 38. The number of movies that

rated PG-13 that earned less than \$100 million was 34. What was the number of movies that earned over \$100 million and that were not rated PG-13? (Data from: www.the-numbers.com.)

19. **Movies** In 2016, among the top 100 grossing movies in the United States, 29 were classified as adventure and earned more than \$75 million. The number of movies that earned over \$75 million was 46. The number of movies that were not adventure movies and earned less than \$75 million was 55. What was the number of movies that were adventure movies that earned less than \$75 million? (Data from: www.the-numbers.com.)

20. **Job Applicants** The human resources director for a commercial real estate company received the following numbers of applications from people with the given information:
- 69 with sales experience;
 - 20 with a college degree;
 - 25 with a real estate license;
 - 26 with sales experience and a college degree;
 - 16 with sales experience and a real estate license;
 - 15 with a college degree and a real estate license;
 - 11 with sales experience, a college degree, and a real estate license;
 - 22 with neither sales experience, a college degree, nor a real estate license.

- (a) How many applicants were there?
- (b) How many applicants did not have sales experience?
- (c) How many had sales experience and a college degree, but not a real estate license?
- (d) How many had only a real estate license?

21. **Product Sales** A pet store keeps track of the purchases of customers over a four-hour period. The store manager classifies purchases as containing a dog product, a cat product, a fish product, or a product for a different kind of pet. She found that:
- 83 customers purchased a dog product;
 - 101 customers purchased a cat product;
 - 22 customers purchased a fish product;
 - 31 customers purchased a dog and a cat product;
 - 8 customers purchased a dog and a fish product;
 - 10 customers purchased a cat and a fish product;
 - 6 customers purchased a dog, a cat, and a fish product;
 - 14 customers purchased a product for a pet other than a dog, cat, or fish.

- (a) How many purchases were for a dog product only?
- (b) How many purchases were for a cat product only?
- (c) How many purchases were for a dog or fish product?
- (d) How many purchases were there in total?

22. **Recreational Fishing** A marine biologist surveys people who fish on Lake Erie and caught at least one fish to determine whether they had caught a walleye, a smallmouth bass, or a yellow perch in the last year. He finds:
- 124 caught at least one walleye;
 - 133 caught at least one smallmouth bass;

146 caught at least one yellow perch;

75 caught at least one walleye and at least one smallmouth bass;

67 caught at least one walleye and at least one yellow perch;

79 caught at least one smallmouth bass and at least one yellow perch;

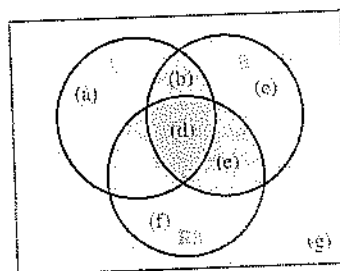
45 caught all three.

- (a) Find the total number of people surveyed.

- (b) How many caught at least one walleye or at least one smallmouth bass?

- (c) How many caught only walleye?

23. **Blood Type** Human blood can contain either no antigens, the A antigen, the B antigen, or both the A and B antigens. A third antigen, called the Rh antigen, is important in human reproduction and, like the A and B antigens, may or may not be present in an individual. Blood is called type A positive if the individual has the A and Rh antigens, but not the B antigen. A person having only the A and B antigens is said to have type AB-negative blood. A person having only the Rh antigen has type O-positive blood. Other blood types are defined in a similar manner. Identify the blood type of the individuals in regions (a)–(g) of the Venn diagram.



23. **Blood Type** Use the diagram from Exercise 22. In a certain hospital, the following data was recorded:

25 patients had the A antigen;

17 had the A and B antigens;

27 had the B antigen;

22 had the B and Rh antigens;

30 had the Rh antigen;

12 had none of the antigens;

16 had the A and Rh antigens;

15 had all three antigens.

How many patients

- (a) were represented?

- (b) had exactly one antigen?

- (c) had exactly two antigens?

- (d) had O-positive blood?

- (e) had AB-positive blood?

- (f) had B-negative blood?

- (g) had O-negative blood?

- (h) had A-positive blood?

24. **Portfolio Management** In reviewing the portfolios of 365 of its clients, a mutual funds company categorized whether the clients were invested in international stock funds, domestic stock funds, or bond funds. It found that,

125 were invested in domestic stocks, international stocks, and bond funds;

145 were invested in domestic stocks and bond funds;

300 were invested in domestic stocks;

200 were invested in international and domestic stocks;

18 were invested in international stocks and bond funds, but not domestic stocks;

35 were invested in bonds, but not in international or domestic stocks;

87 were invested in international stocks, but not in bond funds.

- How many were invested in international stocks?
- How many were invested in bonds, but not international stocks?
- How many were not invested in bonds?
- How many were invested in international or domestic stocks?

For Exercises 25–30, use the given contingency tables. (See Example 7.)

Partnerships In Exercises 25 and 26, the given contingency table lists the cross tabulation of the number of business partnerships by industry and whether the partnership reported net income or net loss for the year 2014. (Data from: U.S. Internal Revenue Service.)

Industry	Number of Partnerships	
	With Net Income (A)	With Net Loss (B)
Construction (C)	75,071	67,561
Manufacturing (M)	24,389	42,386
Wholesale Trade (W)	37,023	45,369
Retail Trade (R)	86,749	81,878

Using the letters given in the table, find the number of partnerships in each set.

- $A \cup C$
 - $M \cup W$
 - $R \cap B$
 - C'
- $B \cup R$
 - $W \cup R$
 - $M \cap A$
 - W'

Life Classification In Exercises 27 and 28, the given contingency table lists the cross tabulation of whether adults (for whom answering the question was appropriate) feel their life is exciting, routine, or dull for males and females among the 1873 appropriate adults in the 2016 General Social Survey. (Data from: gss.norc.umd.edu.)

Life Classification	Males (M)	Females (F)
Exciting (E)	435	510
Routine (R)	357	478
Dull (D)	38	55

Using the letters in the table, find the number of respondents in each set.

- E
 - $D \cap F$
 - $E \cup M$
 - E'
- D
 - $R \cap M$
 - $D \cup F$
 - R'

Job Satisfaction In Exercises 29–32, the given contingency table lists the cross tabulation of how satisfied or dissatisfied adults (for whom answering the question was appropriate) are with their job for males and females among the 933 appropriate adults in the 2016 General Social Survey. (Data from: gss.norc.umd.edu.)

Job Satisfaction	Males (M)	Females (F)
Completely Satisfied (CS)	86	78
Very Satisfied (VS)	176	167
Fairly Satisfied (FS)	149	144
Neither Satisfied nor Dissatisfied (N)	26	28
Fairly Dissatisfied (FD)	20	32
Very Dissatisfied (VD)	9	11
Completely Dissatisfied (CD)	3	4

Using the letters given in the table, find the number of respondents in each set.

- $CS \cup VS$
 - $(CS \cup VS) \cap F$
 - Describe your answer to part (b) in words.
- $VD \cup CD$
 - $(VD \cup CD) \cap M$
 - Describe your answer to part (b) in words.
- $CS \cup VS \cup FS$
 - $(CS \cup VS \cup FS) \cap M$
 - Describe your answer to part (b) in words.
- $FD \cup VD \cup CD$
 - $(FD \cup VD \cup CD) \cap F$
 - Describe your answer to part (b) in words.

33. Restate the union rule in words.

Use Venn diagrams to answer the given questions. (See Example 6.)

- If $n(A) = 5$, $n(B) = 8$, and $n(A \cap B) = 4$, what is $n(A \cup B)$?
- If $n(A) = 12$, $n(B) = 27$, and $n(A \cup B) = 30$, what is $n(A' \cap B)$?
- Suppose $n(B) = 7$, $n(A \cap B) = 3$, and $n(A \cup B) = 20$. What is $n(A)$?
- Suppose $n(A \cap B) = 5$, $n(A \cup B) = 35$, and $n(A) = 13$. What is $n(B)$?

Draw a Venn diagram and use the given information to fill in the number of elements for each region.

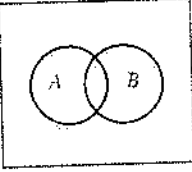
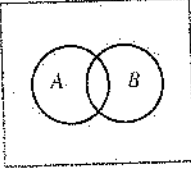
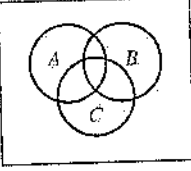
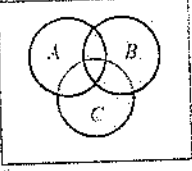
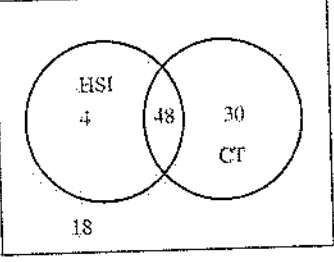
38. $n(U) = 48, n(A) = 26, n(A \cap B) = 12, n(B') = 30$
39. $n(U) = 28, n(B) = 12, n(A \cup B) = 30, n(A') = 19$
40. $n(A \cap B) = 17, n(A \cap B) = 3, n(A) = 8, n(A' \cup B') = 21$
41. $n(U) = 28, n(B) = 25, n(A' \cup B') = 45, n(A \cap B) = 12$
42. $n(U) = 28, n(B) = 34, n(C) = 25, n(A \cap B) = 14, n(B \cap C) = 25, n(A \cap C) = 11, n(A \cap B \cap C) = 9, n(U) = 59$
43. $n(U) = 58, n(A \cap B) = 22, n(A \cup B) = 85, n(A \cap B \cap C) = 4, n(A \cap C) = 15, n(B \cap C) = 16, n(C) = 44, n(B') = 63$

Exercises 44–47: show that the statements are true by drawing Venn diagrams and shading the regions representing the sets on each side of the equals sign.*

44. $(A \cap B)' = A' \cup B'$
45. $(A \cup B)' = A' \cap B'$
46. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
47. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
48. Explain in words the statement about sets in question 44.
49. Explain in words the statement about sets in question 45.
50. Explain in words the statement about sets in question 46.
51. Explain in words the statement about sets in question 47.

*The exercises in Exercises 44 and 45 are known as De Morgan's laws. They are named after the English mathematician Augustus De Morgan (1806–1871).

Checkpoint Answers

1. (a)  $A \cup B'$
(b)  $A' \cap B'$
2. (a)  $(B' \cap A) \cup C$
(b)  $(A \cup B)' \cap C$
(b) 4
3. (a)  (b) 9
4. (a) 42 (b) 9
5. (a) 8 (b) 92 (c) 98
6. 14
7. (a) 151,437 thousand
(b) 84,435 thousand

8.3 Introduction to Probability

If you go to a pizzeria and order two large pizzas at \$14.99 each, you can easily find the *exact* price of your purchase: \$29.98. For the manager at the pizzeria, however, it is impossible to predict the *exact* number of pizzas to be purchased daily. The number of pizzas purchased during a day is *random*. The quantity cannot be predicted exactly. A great many problems that come up in applications of mathematics involve random phenomena—phenomena for which exact prediction is impossible. The best that we can do is determine the *probability* of the possible outcomes.

Random Experiments and Sample Spaces

A **random experiment** (sometimes called a random phenomenon) has outcomes that we cannot predict, but that nonetheless have a regular distribution in a large number of repetitions. We call a repetition from a random experiment a **trial**. The possible results of each trial are called **outcomes**. For instance, when we flip a coin, the outcomes are heads and tails. We do not know whether a particular flip will yield heads or tails, but we do know that if we flip the coin a large number of times, about half the flips will be heads and half will be tails. Each flip of the coin is a trial. The **sample space** (denoted by S) for a random experiment is the set of all possible outcomes. For the coin flipping, the sample space is

$$S = \{\text{heads, tails}\}.$$

8.3 Exercises

1. What is meant by a "fair" coin or die?
 2. What is the sample space for a random experiment?

Write sample spaces for the random experiments in Exercises 3–9.

Example 1.3

3. A month of the year is chosen for a wedding.
 4. A date in April is selected for a bicycle race.
 5. A student is asked how many points she earned on a recent report test.
 6. A person is asked the number of hours (to the nearest hour) he watched television yesterday.
 7. The management of an oil company must decide whether to go ahead with a new oil shale plant or to cancel it.
 8. A coin is tossed and a die is rolled.
 9. The quarter of the year in which a company's profits were highest.
10. Define an event.
 11. Define disjoint events in your own words.

Decide whether the events are disjoint. (See Example 5.)

12. Owning an SUV and owning a Jeep
13. Wearing a hat and wearing glasses
14. Being married and being under 30 years old
15. Being a doctor and being under 5 years old
16. Being male and being a nurse
17. Being female and being a pilot

For the random experiments in Exercises 18–20, write out an equally likely sample space, and then write the indicated events in set notation. (See Examples 2 and 3.)

18. A marble is drawn at random from a bowl containing 3 yellow, 4 white, and 8 blue marbles.
 - (a) A yellow marble is drawn.
 - (b) A blue marble is drawn.
 - (c) A white marble is drawn.
 - (d) A black marble is drawn.
19. Six people live in a dorm suite. Two are to be selected to go to the campus café to pick up a pizza. Of course, no one wants to go, so the six names (Connie, Kate, Lindsey, Jackie, Taisa, and Nicole) are placed in a hat. After the hat is shaken, two names are selected.
 - (a) Taisa is selected.
 - (b) The two names selected have the same number of letters.
20. An unprepared student takes a three-question true-or-false quiz in which he flips a coin to guess the answers. If the coin is heads, he guesses true, and if the coin is tails, he guesses false.

- (a) The student guesses true twice and guesses false once.
- (b) The student guesses all false.
- (c) The student guesses true once and guesses false twice.

In Exercises 21–24, write out the sample space and assume each outcome is equally likely. Then give the probability of the requested outcomes. (See Examples 7 and 8.)

21. In deciding what color and style to paint a room, Greg has narrowed his choices to three colors—forest sage, evergreen whisper, and opaque emerald—and two styles—rag painting and colorwash.
 - (a) Greg picks a combination with colorwash.
 - (b) Greg picks a combination with opaque emerald or rag painting.
22. Tami goes shopping and sees three kinds of shoes: flats, 2" heels, and 3" heels. They come in two shades of beige (light and dark) and black.
 - (a) The shoe selected has a heel and is black.
 - (b) The shoe selected has no heel and is beige.
 - (c) The shoe selected has a heel and is beige.
23. Doug is shopping for a new patio umbrella. There is a 10-foot and a 12-foot model, and each is available in beige, forest green, and rust.
 - (a) Doug buys a 12-foot forest green umbrella.
 - (b) Doug buys a 10-foot umbrella.
 - (c) Doug buys a rust-colored umbrella.
24. Sharonda is deciding between four brands of cell phones: Samsung, Motorola, Nokia, and Sony. Each of these phones comes with the option of insurance or no insurance.
 - (a) Sharonda picks a phone with insurance.
 - (b) Sharonda picks a phone that begins with the letter "S" and no insurance.

A single fair die is rolled. Find the probabilities of the given events. (See Example 6.)

25. Getting a number less than 4
26. Getting a number greater than 4
27. Getting a 2 or a 5
28. Getting a multiple of 3

Alejandro wants to adopt a puppy from an animal shelter. At the shelter, he finds eight puppies that he likes: a male and female puppy from each of the four breeds of beagle, boxer, collie, and Labrador. The puppies are each so cute that Alejandro cannot make up his mind, so he decides to pick the dog randomly.

29. Write the sample space for the outcomes, assuming each outcome is equally likely.

Find the probability that Alejandro chooses the given puppy.

30. A male boxer
31. A male puppy

32. A collie
34. A beagle or a boxer
36. Anything except a male beagle or male boxer.
33. A female Labrador
35. Anything except a Labrador

Fatal Work Injuries The following table gives the number of fatal work injuries categorized by cause from 2015. (Data from: U.S. Bureau of Labor Statistics.)

Cause	Number of Fatalities
Transportation accidents	2054
Assaults and violent acts	703
Contacts with objects and equipment	722
Falls	800
Exposure to harmful substances or environments	424
Fires and explosions	121
Other	12

Find the probability that a randomly chosen work fatality had the given cause. (See Example 8.)

37. A fall
38. Fires and explosions
39. Religious Services Respondents to the 2016 General Social Survey (GSS) indicated the following categorizations pertaining to attendance at religious services: (Data from: gss.norc.umd.edu.)

Attendance	Number of Respondents
Never	723
Less than once a year	173
Once a year	385
Several times a year	312
Once a month	190
2–3 times a month	250
Nearly every week	126
Every week	481
More than once a week	210
Don't know/no answer	17
Total	2867

Find the probability that a randomly chosen person in the United States attends religious services

- (a) several times a year;
(b) 2–3 times a month;
(c) nearly every week or more frequently.

40. Gender Roles Respondents to the 2016 General Social Survey (GSS) indicated whether they strongly agreed, agreed, disagreed, or strongly disagreed with the question, “It is better for

men to work and for women to tend to the home.” The responses are categorized in the following table. (Data from: gss.norc.umd.edu.)

Response	Number of Respondents
Strongly Agree	110
Agree	405
Disagree	853
Strongly Disagree	504
Don't Know/No Answer	995

Find the probability that a person in the United States:

- (a) strongly agreed or agreed with the statement;
(b) strongly disagreed with the statement;
(c) did not strongly agree with the statement.

S&P 500 As of March 7, 2017, the 505 stocks in the S&P 500 were classified in 11 different sectors. The following table gives the numbers of stocks in each sector. (Data from: www.morningstar.com.)

Sector	Number of Stocks in Sector
Consumer Discretionary	86
Consumer Staples	37
Energy	35
Financials	65
Health Care	60
Industrials	66
Information Technology	68
Materials	25
Real Estate	30
Telecommunications Services	5
Utilities	28
Total	505

Find the probability that a stock in the S&P 500 is in the requested sector:

41. Energy
42. Financials
43. Real Estate or Utilities
44. Health Care or Information Technology

S&P 500 The following table gives the counts for three price categories of the 505 stocks in the S&P 500. (Data from: www.morningstar.com.)

Price	Count
\$0–\$49.99	159
\$50–\$99.99	207
\$100 or higher	139

Find the probability that a stock in the S&P 500 is in the requested price range.

45. Below \$50

46. Below \$100

An experiment is conducted for which the sample space is $S = \{s_1, s_2, s_3, s_4, s_5\}$. Which of the probability assignments in Exercises 47–52 is possible for this experiment? If an assignment is not possible, tell why.

Outcomes	s_1	s_2	s_3	s_4	s_5
Probabilities	.09	.32	.21	.25	.13

Outcomes	s_1	s_2	s_3	s_4	s_5
Probabilities	.92	.03	0	.02	.03

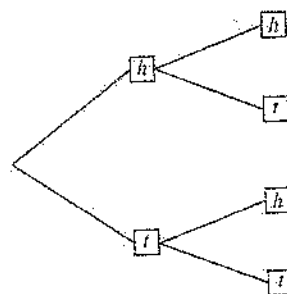
Outcomes	s_1	s_2	s_3	s_4	s_5
Probabilities	$1/3$	$1/4$	$1/6$	$1/8$	$1/10$

Outcomes	s_1	s_2	s_3	s_4	s_5
Probabilities	$1/5$	$1/3$	$1/4$	$1/5$	$1/10$

Outcomes	s_1	s_2	s_3	s_4	s_5
Probabilities	.64	-.08	.30	.12	.02

Outcomes	s_1	s_2	s_3	s_4	s_5
Probabilities	.05	.35	.5	.2	-.3

Checkpoint Answers



- $\{hh, ht, th, tt\}$
- (a) $\{1, 2\}$ (b) $\{5\}$ (c) $\emptyset$
- (a) None (b) Part (c)
- (a) $\{3\}$ (b) $\{1, 2, 3, 4, 5\}$ (c) $\{2, 4, 5, 6\}$
- (a) Yes (b) No
- (a) $1/2$ (b) $2/3$ (c) $1/6$ (d) 0
- (a) $1/13$ (b) $1/4$ (c) $1/2$
- About .8251

8.4 Basic Concepts of Probability

We determine the probability of more complex events in this section.

To find the probability of the union of two sets E and F in a sample space S , we use the union rule for counting given in Section 8.2:

$$n(E \cup F) = n(E) + n(F) - n(E \cap F).$$

Dividing both sides by $n(S)$ yields

$$\frac{n(E \cup F)}{n(S)} = \frac{n(E)}{n(S)} + \frac{n(F)}{n(S)} - \frac{n(E \cap F)}{n(S)}$$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

This discussion is summarized in the next rule.

Addition Rule for Probability

For any events E and F from a sample space S ,

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

In words, we have

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F).$$

7. The marble will land in a slot numbered 25–36.

Two dice are rolled. Find the probabilities of rolling the given sums. (See Examples 2, 4, and 5.)

8. (a) 2 (b) 4 (c) 5 (d) 6
 9. (a) 8 (b) 9 (c) 10 (d) 13
 10. (a) 9 or more
 (b) Less than 7
 (c) Between 5 and 8 (exclusive)
 11. (a) Not more than 5
 (b) Not less than 8
 (c) Between 3 and 7 (exclusive)

Tami goes shopping and sees three kinds of shoes: flats, 2" heels, and 3" heels. The shoes come in two shades of beige (light and dark) and black. If each option has an equal chance of being selected, find the probabilities of the given events.

12. The shoes Tami buys have a heel.
 13. The shoes Tami buys are black.
 14. The shoes Tami buys have a 2" heel and are beige.

Ms. Elliott invites 10 relatives to a party: her mother, 3 aunts, 2 uncles, 2 sisters, 1 male cousin, and 1 female cousin. If the chances of any one guest arriving first are equally likely, find the probabilities that the given guests will arrive first.

15. (a) A sister or an aunt
 (b) A sister or a cousin
 (c) A sister or her mother
 16. (a) An aunt or a cousin
 (b) A male or an uncle
 (c) A female or a cousin

Use Venn diagrams to work Exercises 17–22. (See Example 9.)

17. Suppose $P(E) = .30$, $P(F) = .51$, and $P(E \cap F) = .19$. Find each of the given probabilities.

- (a) $P(E \cup F)$ (b) $P(E' \cap F)$
 (c) $P(E \cap F')$ (d) $P(E' \cup F')$

18. Let $P(Z) = .40$, $P(Y) = .30$, and $P(Z \cup Y) = .58$. Find each of the given probabilities.

- (a) $P(Z' \cap Y')$ (b) $P(Z' \cup Y')$
 (c) $P(Z' \cup Y)$ (d) $P(Z \cap Y')$

19. Fatal Vehicle Accidents According to 2015 data from the National Highway Traffic Safety Administration, the probability that the driver in a fatal vehicle accident is female (event F) is .2584. The probability that the driver is 24 years old or less (event A) is .1914. The probability that the driver is female and is 24 years old or less is .0536. Find the probability of the given events.

- (a) $F \cup A$ (b) $F' \cup A$
 (c) $F \cap A'$ (d) $F' \cup A'$

20. Fatal Vehicle Accidents According to 2015 data from the National Highway Traffic Safety Administration, the probability that an airbag was deployed (event A) during an accident is .4364.

The probability that the driver was impaired (event I) is .1. The probability that the airbag was deployed and the driver impaired is .0778. Find the probability of the given events.

- (a) $A \cup I$ (b) $A \cup I'$
 (c) $A \cap I'$ (d) $A' \cap I'$

21. Marriage and Happiness According to the 2016 General Social Survey, the probability that a U.S. resident is currently married is .4227 and the probability that a U.S. resident describes himself or herself as very happy is .2820. The probability of being married and being very happy is .1627. Find the probability of the described events. (Data from: gss.norc.umd.edu)

- (a) Not being very happy and being married
 (b) Not being married and not being very happy
 (c) Not being married or not being very happy

22. Education and Earnings According to the 2016 General Social Survey, the probability that a U.S. resident has attended junior college or higher is .3845, and the probability that a household income is \$50,000 or higher is .5040. The probability of having attended junior college or higher and being in a household with income \$50,000 or higher is .2700. Find the probability of the described events. (Data from: gss.norc.umd.edu)

- (a) Not having attended junior college or higher and being in a household earning \$50,000 or more.
 (b) Having attended junior college or higher or not being in a household earning \$50,000 or more.
 (c) Not having attended junior college and not being in a household earning \$50,000 or more.

A single fair die is rolled. Find the odds in favor of getting the results in Exercises 23–26. (See Examples 6 and 7.)

23. 2 24. 2, 3, 4
 25. 2, 3, 5, or 6 26. Some number greater than 3
 27. A marble is drawn from a box containing 3 yellow, 4 white, and 5 blue marbles. Find the odds in favor of drawing the given marble.
 (a) A yellow marble
 (b) A blue marble
 (c) A white marble

28. Find the odds of not drawing a white marble in Exercise 27.

29. Two dice are rolled. Find the odds of rolling a 7 or an 11.

30. Define what is meant by odds.

Education For Exercises 31–34, find the odds of the event occurring from the given probability that a bachelor's degree recipient in 2013 majored in the given discipline. (Data from: *Digest of Education Statistics*.)

31. Business; probability 19/100.
 32. Natural sciences and mathematics; probability 2/25.
 33. Education; probability 1/20.
 34. Social and behavioral sciences; 3/20.

Government Revenue For Exercises 35–38, convert the given odds to the probability of the event. The events involve revenue for the U.S. government from taxes. (Data from: U.S. Census Bureau.)

40. The odds that tax revenue comes from property taxes are 11 to 9.
41. The odds that tax revenue comes from sales and gross receipts are 7 to 13.
42. The odds that tax revenue comes from individual income tax are 25 to 7.
43. The odds that tax revenue comes from corporate income tax are 7 to 605.

500 Stocks For Exercises 39–46, use the following table, which classifies the price of the 505 stocks in the S&P 500 on March 7, 2017, with whether the earnings per share ratio was positive or not.

Price of Stock	Earnings per Share	
	Negative or 0	Positive
\$0–\$49.99	30	128
\$50.00–\$99.99	18	189
\$100.00 or higher	5	135

Find the probability of the described events. (Data from: www.okfn.org.)

39. The earnings per share ratio is positive.
40. The earnings per share ratio is not positive.
41. The price of the stock is less than \$50 and the earnings per share ratio is negative or 0.
42. The price of the stock is \$100 or higher and the earnings per share ratio is positive.
43. The price of the stock is \$50.00–\$99.99 or the earnings per share ratio is positive.
44. The price of the stock is \$100 or higher or the earnings per share ratio is positive.
45. The price of the stock is not below \$50.00 and the earnings per share ratio is positive.
46. The price of the stock is not \$100 or higher and the earnings per share ratio is negative or 0.

Hours Worked Data from the 2016 General Social Survey can allow us to estimate how many hours people who classified their work status as part-time or full-time actually worked in the last week. Use the following table to find the probabilities of the events in Exercises 47–54. (Data from: gss.norc.umd.edu.)

Labor Force Status	Hours Worked in the Last Week					Total
	0–19	20–29	30–39	40–49	50 or More	
Full-Time Worker	26	43	137	691	377	1274
Part-Time Worker	74	82	87	41	24	308
Total	100	125	224	732	401	1582

47. Full-time worker
48. Part-time worker
49. Worked 50 hours or more
50. Worked 0–19 hours
51. Part-time worker and someone who worked 40 hours or more
52. Full-time worker and someone who worked less than 40 hours
53. Full-time worker or someone who worked 40 hours or more
54. Part-time worker or someone who worked 29 hours or less

One way to solve a probability problem is to repeat the experiment many times, keeping track of the results. Then the probability can be approximated by using the basic definition of the probability of an event E , which is $P(E) = n(E)/n(S)$, where E occurs $n(E)$ times out of $n(S)$ trials of an experiment. This is called the Monte Carlo method of finding probabilities. If physically repeating the experiment is too tedious, it may be simulated with the use of a random-number generator, available on most computers and scientific or graphing calculators. To simulate a coin toss or the roll of a die on a graphing calculator, change the setting to fixed decimal mode with 0 digits displayed. To simulate multiple tosses of a coin, press **RAND** (or **RANDOM** or **RND#**) in the **PROB** submenu of the **MATH** (or **OPTN**) menu, and then press **ENTER** repeatedly. Interpret 0 as a head and 1 as a tail. To simulate multiple rolls of a die, press **RAND** $\times$ 6 $\div$.5, and then press **ENTER** repeatedly.

55. Suppose two dice are rolled. Use the Monte Carlo method with at least 50 repetitions to approximate the given probabilities. Compare them with the results of Exercise 10.
- (a) $P(\text{the sum is 9 or more})$
- (b) $P(\text{the sum is less than 7})$
56. Suppose two dice are rolled. Use the Monte Carlo method with at least 50 repetitions to approximate the given probabilities. Compare them with the results of Exercise 11.
- (a) $P(\text{the sum is not more than 5})$
- (b) $P(\text{the sum is not less than 8})$
57. Suppose three dice are rolled. Use the Monte Carlo method with at least 100 repetitions to approximate the given probabilities.
- (a) $P(\text{the sum is 5 or less})$
- (b) $P(\text{neither a 1 nor a 6 is rolled})$
58. Suppose a coin is tossed 5 times. Use the Monte Carlo method with at least 50 repetitions to approximate the given probabilities.
- (a) $P(\text{exactly 4 heads})$
- (b) $P(2 \text{ heads and 3 tails})$

Genetics Gregor Mendel, an Austrian monk, was the first to use probability in the study of genetics. In an effort to understand the mechanism of characteristic transmission from one generation to the next in plants, he counted the number of occurrences of various characteristics. Mendel found that the flower color in certain pea plants obeyed this scheme:

Pure red crossed with pure white produces red.

From its parents, the red offspring received genes for both red (R) and white (W), but in this case red is dominant and white recessive, so the offspring exhibits the color red. However, the offspring still carries both

8.5 Exercises

If a single fair die is rolled, find the probability of rolling the given events. (See Examples 1 and 2.)

- 3, given that the number rolled was odd
- 5, given that the number rolled was even
- An odd number, given that the number rolled was 3

■ If two fair dice are rolled (recall the 36-outcome sample space), find the probability of rolling the given events.

- A sum of 8, given that the sum was greater than 7
- A sum of 6, given that the roll was a "double" (two identical numbers)
- A double, given that the sum was 9

■ If two cards are drawn without replacement from an ordinary deck, find the probabilities of the given event. (See Example 7.)

- The second is a heart, given that the first is a heart
- The second is black, given that the first is a spade
- A jack and a 10 are drawn.
- An ace and a 4 are drawn.

- In your own words, explain how to find the conditional probability $P(E|F)$.
- Your friend asks you to explain how the product rule for independent events differs from the product rule for dependent events. How would you respond?
- Another friend asks you to explain how to tell whether two events are dependent or independent. How would you reply? (Use your own words.)
- A student reasons that the probability in Example 3 of both coins being heads is just the probability that the other coin is a head—that is, $1/2$. Explain why this reasoning is wrong.

Decide whether the two events listed are independent.

- S is the event that it is snowing and L is the event that the instructor is late for class.
- R is the event that four semesters of theology are required to graduate from a certain college and A is the event that the college is a religiously affiliated school.
- R is the event that it rains in the Amazon jungle and H is the event that an instructor in New York City writes a difficult exam.
- T is the event that Tom Cruise's next movie grosses over \$200 million and R is the event that the Republicans have a majority in Congress in the year 2024.
- In a two-child family, if we assume that the probabilities of a child being male and a child being female are each .5, are the events "the children are the same sex" and "at most one child is male" independent? Are they independent for a three-child family?

- Let A and B be independent events with $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{5}$. Find $P(A \cap B)$ and $P(A \cup B)$.

Firm Size According to data from the U.S. Small Business Administration, the probability that a firm has one to four employees given that it is a construction firm is .6930. The probability that a firm has one to four employees given that it is a manufacturing firm is .4196. The probability the firm is a construction firm is .1135, and the probability that a firm is a manufacturing firm is .0449. The probability that a firm has one to four employees is .6214. Use this information to find the probabilities for Exercises 21–24. (See Example 4.)

- A firm has one to four employees and is a construction firm.
- A firm has one to four employees and is a manufacturing firm.
- A firm is a construction firm given that it has one to four employees.
- A firm is a manufacturing firm given that it has one to four employees.

Hours Worked Data from the 2016 General Social Survey can allow us to estimate how many hours people who classified their work status as part-time or full-time actually worked in the last week. Use the following table to find the probabilities of the events in Exercises 25–30. (Data from: gss.norc.berkeley.edu.)

Labor Force Status	Hours Worked in the Last Week					Total
	0–19	20–29	30–39	40–49	50 or More	
Full-Time Worker	26	43	137	691	377	1274
Part-Time Worker	74	82	87	41	24	208
Total	100	125	224	732	401	1582

- The worker was full-time given that he or she worked 50 or more hours.
- The worker was part-time given that she or he worked 50 or more hours.
- The hours worked were 0–19 given that the worker was full-time.
- The hours worked were 20–29 given that the worker was part-time.
- The hours worked were 20 or less given that the worker was part-time.
- The hours worked were 40 or more given that the worker was full-time.

Consumer Confidence The 2016 General Social Survey asked respondents about their confidence in banks and financial institutions as well as their confidence in major corporations. Each question allowed three responses: "a great deal, only some, or hardly any." The table below summarizes the responses.

Confidence in Banks and Financial Institutions	Confidence in Major Corporations			Total
	A Great Deal	Only Some	Hardly Any	
A Great Deal	125	119	25	269
Only Some	172	759	113	1044
Hardly Any	43	362	201	606
Total	340	1240	339	1919

Find the probability of the described events. (Data from: gss.norc.umd.edu.)

A respondent has a great deal of confidence in banks and financial institutions given that he or she has hardly any confidence in major corporations.

A respondent has hardly any confidence in banks and financial institutions given that he or she has hardly any confidence in major corporations.

A respondent has a great deal of confidence in major corporations given that he or she has only some confidence in banks and financial institutions.

A respondent has hardly any confidence in major corporations given that he or she has hardly any confidence in banks and financial institutions.

Color Blindness The following table shows frequencies for red-green color blindness, where M represents that a person is male and C represents that a person is color blind.

	M	M'	Totals
C	.042	.007	.049
C'	.485	.466	.951
Totals	.527	.473	1.000

Use the table to find the given probabilities.

36. $P(M)$

38. $P(M \cup C)$

40. $P(M'|C)$

Are the events C and M dependent? Recall that two events E and F are dependent if $P(E|F) \neq P(E)$. (See Example 10.)

Are the events M' and C dependent?

Color Blindness and Deafness A scientist wishes to determine whether there is any dependence between color blindness (C) and deafness (D). Use the probabilities in the table to answer Exercises 43 and 44.

	D	D'	Total
C	.0004	.0796	.0800
C'	.0046	.9154	.9200
Total	.0050	.9950	1.0000

43. Find $P(C)$, $P(D)$, $P(C \cap D)$, and $P(C|D)$. Does $P(C) = P(C|D)$? What does that result imply regarding independence of C and D ?

44. Find $P(D')$, $P(C)$, $P(D' \cap C)$, and $P(D'|C)$. Does $P(D') = P(D'|C)$? What does that result imply regarding independence of D' and C ?

Driver's Test The Motor Vehicle Department in a certain state has found that the probability of a person passing the test for a driver's license on the first try is .75. The probability that an individual who fails on the first test will pass on the second try is .80, and the probability that an individual who fails the first and second tests will pass the third time is .70. Find the probability of the given event.

45. A person fails both the first and second tests

46. A person will fail three times in a row

Airlines The number of on-time flights and the number of delayed or cancelled flights for four major airlines for March 2017 is given in the table below.

Airline	On Time	Delayed or Cancelled
American	62,642	15,472
Delta	68,721	10,349
Southwest	91,664	23,480
United	39,262	9235

Find the probability of the described event. (Data from: www.transportation.gov.)

47. An on-time flight given the airline was American.

48. An on-time flight given the airline was Delta.

49. An on-time flight given the airline was Southwest.

50. An on-time flight given the airline was United.

51. The airline was United given that the flight was on time.

52. The airline was American given that the flight was on time.

53. The airline was not Delta given that the flight was delayed or cancelled.

54. The airline was not American given that the flight was delayed or cancelled.

55. Which airline had the highest probability of an on-time flight?

56. Which airline had the lowest probability of an on-time flight?

Suppose the probability that the first record by a singing group will be a hit is .32. If the first record is a hit, so are all the group's subsequent records. If the first record is not a hit, the probability of the group's second record and all subsequent ones being hits is .16. If the first two records are not hits, the probability that the third is a hit is .08. The probability that a record is a hit continues to decrease by half with each successive nonhit record. Find the probability of the given event.

57. The group will have at least one hit in its first four records.

58. The group will have a hit in its first six records if the first three are not hits.

Work the given problems on independent events. (See Examples 9 and 10.)

59. **Computer Failure** Corporations such as banks, where a computer is essential to day-to-day operations, often have a second, backup computer in case of failure by the main computer. Suppose that there is a .003 chance that the main computer will fail in a given period and a .005 chance that the backup computer will fail while the main computer is being repaired. Suppose these failures represent independent events, and find the fraction of the time the corporation can assume that it will have computer service.
60. **Multiple Flights** According to data from the U.S. Department of Transportation, Delta Airlines was on time approximately 87% of the time in March 2017. Use this information, and assume that the event that a given flight takes place on time is independent of the event that another flight is on time to answer the following questions.
- Elisabeta plans to visit her company's branch offices; her journey requires 3 separate flights on Delta Airlines. What is the probability that all of these flights will be on time?
 - How reasonable do you believe it is to suppose the independence of being on time from flight to flight?
61. **Rocket Failure** The probability that a key component of a space rocket will fail is .03.
- How many such components must be used as backups to ensure that the probability that at least one of the components will work is .999999?
 - Is it reasonable to assume independence here?

62. **Medication Effectiveness** A medical experiment shows that the probability that a new medicine is effective is .75, the probability that a patient will have a certain side effect is .4, and the probability that both events will occur is .3. Decide whether these events are dependent or independent.

Consumer Confidence Refer to the table included for Exercise 31–34.

- Are the events of having hardly any confidence in banks and financial institutions and having hardly any confidence in major corporations independent?
- Are the events of having a great deal of confidence in banks and financial institutions and having a great deal of confidence in major corporations independent?

Checkpoint Answers

- (a) About .0891 (b) About .5743 (c) About .7228
- (a) About .3849 (b) About .1943 (c) About .7488 (d) About .4655 (e) The probability of never being married, given that the person is partially or not happy
- 1/3 4. 2/3 5. 2/7
- (a) 1/4 (b) 3/5
- 1/5 8. 1/10 9. $13/102 \approx .1275$
- $13/34 \approx .3824$ 11. 1/16 12. No

8.6 Bayes' Formula

Suppose the probability that a person gets lung cancer, given that the person smokes a pack or more of cigarettes daily, is known. For a research project, it might be necessary to know the probability that a person smokes a pack or more of cigarettes daily, given that the person has lung cancer. More generally, if $P(E|F)$ is known for two events E and F , can $P(F|E)$ be found? The answer is yes; we can find $P(F|E)$ by using the formula to be developed in this section. To develop this formula, we can use a probability tree to find $P(F|E)$. Since $P(E|F)$ is known, the first outcome is either F or F' . Then, for each of these outcomes, either E or E' occurs, as shown in Figure 8.33.

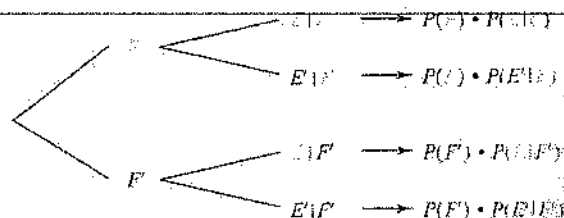


Figure 8.33

The four cases have the probabilities shown on the right. By the definition of conditional probability and the product rule,

$$P(E) = P(F \cap E) + P(F' \cap E),$$

$$P(F|E) = \frac{P(F \cap E)}{P(E)}, \text{ and } P(F'|E) = \frac{P(F' \cap E)}{P(E)}$$

Solution Let F_1 be the event that an item came from supplier 1, with F_2, F_3, F_4, F_5 , and F_6 defined in a similar manner. Let E be the event that an item is defective. We want to find $P(F_5|E)$. By Bayes' formula,

$$P(F_5|E) = \frac{(.35)(.03)}{(.05)(.04) + (.12)(.02) + (.16)(.07) + (.23)(.01) + (.35)(.03) + (.09)(.05)} \\ = \frac{.0105}{.0329} \approx .3191.$$

Checkpoint 4

In Example 3, find the probability that the defective item came from

- supplier 3;
- supplier 6.

There is about a 32% chance that a defective item came from supplier 5. Even though supplier 5 has only 3% defectives, his probability of being "guilty" is relatively high, about 32%, because of the large fraction of items he supplies. $\square$

8.6 Exercises

For two events M and N , $P(M) = .4$, $P(N|M) = .3$, and $P(N|M') = .5$.
4. Find each of the given probabilities. (See Example 1.)

- $P(M|N)$
- $P(M'|N)$

For disjoint events R_1, R_2 , and R_3 , $P(R_1) = .05$, $P(R_2) = .6$, and $P(R_3) = .35$. In addition, $P(Q|R_1) = .40$, $P(Q|R_2) = .30$, and $P(Q|R_3) = .60$. Find each of the given probabilities. (See Examples 2 and 3.)

- $P(R_1|Q)$
- $P(R_2|Q)$
- $P(R_3|Q)$
- $P(R_1'|Q)$

Suppose three jars have the following contents: 2 black balls and 1 white ball in the first; 1 black ball and 2 white balls in the second; 1 black ball and 1 white ball in the third. If the probability of selecting one of the three jars is $1/2$, $1/3$, and $1/6$, respectively, find the probability that if a white ball is drawn, it came from the given jar.

- The second jar.
- The third jar.

S&P 500 Stocks On March 7, 2017, the probability that a stock was in the Information Technology sector was .1347. The probability that the stock had a dividend yield of 2.00% or higher given that it was in the Information Technology sector was .2941. The probability that a stock had a dividend yield of 2.00% or higher given that it was not in the Information Technology sector was .4462. Find the following probabilities. (Data from: www.okfn.org.)

- The stock was in the Information Technology sector given that it had a dividend yield of 2.00% or higher.
- The stock was not in the Information Technology sector given that it did not have a dividend yield of 2.00% or higher.

Educational Attainment and Earnings Data from the U.S. Census Bureau indicates that for the year 2015, 33.3% of the labor force age 25 and over had a high school diploma or fewer years of education, 27.9% had some college or an associate's degree, and 38.9% had a bachelor's degree or more education. Of those with a high school diploma or fewer years of education, 7.4% earned \$75,000 or more. Of those with some college or an associate's degree, 13.5% earned \$75,000 or more, and of those with a bachelor's degree or more education, 37.0% earned \$75,000 or more. Find the following probabilities that a randomly chosen

labor force participant has the given characteristics. (Data from: www.census.gov.)

- Some college or an associate's degree, given that he or she earns \$75,000 or more annually
- A bachelor's degree or more education, given that he or she earns \$75,000 or more annually
- A high school diploma or less education, given that he or she earns less than \$75,000 annually
- A bachelor's degree or more education, given that he or she earns less than \$75,000 annually

Airlines In March 2017, the probability that a flight was an American Airlines flight was .1599, the probability that it was a Delta flight was .1618, and the probability that it was another airline was .6783. The probability that the flight was on time given that it was an American Airlines flight was .8019, given that it was a Delta flight was .8691, and given that it was another airline was .7820. Find the probability of the following events. (Data from: www.transportation.gov.)

- American Airlines given that it was on time.
- Delta given that it was on time.
- An airline not American or Delta given that it was not on time.
- Delta given that it was not on time.

Health In a test for toxemia, a disease that affects pregnant women, a woman lies on her left side and then rolls over on her back. The test is considered positive if there is a 20-mm rise in her blood pressure within 1 minute. The results have produced the following probabilities, where T represents having toxemia at some time during the pregnancy and N represents a negative test:

$$P(T|N) = .90 \quad \text{and} \quad P(T|N') = .75.$$

Assume that $P(N') = .11$, and find each of the given probabilities.

- $P(N|T)$
- $P(N'|T)$

Business In April 2017, the total sales from General Motors, Ford, or Chrysler was 630,596 cars or light trucks. The probability that the vehicle sold was made by General Motors was .3873, by Ford .3385, and by Chrysler .2742. Additionally, the probability that a General Motors

which sold was a car was .2765, a Ford vehicle sold was a car was .2306, and a Chrysler vehicle sold was a car was .1253.

11. Given the vehicle sold was a car, find the probability it was made by General Motors.
12. Given the vehicle sold was a car, find the probability it was made by Chrysler.
13. Given the vehicle sold was not a car, find the probability it was made by Ford.
14. Given the vehicle sold was not a car, find the probability it was made by General Motors.

Business At the close of the markets on May 19, 2017, there were 490 companies listed with the New York Stock Exchange (NYSE) and NASDAQ. For the day May 19, 2017, based on data from the Wall Street Journal, the probabilities of advancing, declining, and unchanged company stocks appears below for the two exchanges.

Exchange	Percentage of Companies Listed	Probability of Stock Advancing	Probability of Stock Declining	Probability of Stock Remaining Unchanged
NYSE	.5085	.7240	.2453	.0306
NASDAQ	.4915	.6024	.3426	.0540

Find the probability of each of the following events.

15. The stock was listed on the NYSE given that it advanced.
16. The stock was listed on NASDAQ given that it declined.
17. The stock was listed on the NYSE given that it remained unchanged.
18. The stock was listed on NASDAQ given that it remained unchanged.

29. The stock was listed on NASDAQ given that it advanced.
30. The stock was listed on the NYSE given that it declined.

Happiness The proportion of adult males is .492 and the proportion of adult females is .508. Data from the 2016 General Social Survey has the following probabilities of happiness for each sex. (Data from: U.S. Census Bureau and [gss.norc.org](http://gss.norc.umd.edu).)

Sex	Proportion	Very Happy	Pretty Happy	Not Too Happy
Male	.492	.287	.564	.149
Female	.508	.277	.558	.165

Find the probability of the described events.

31. Given that the person is very happy, the respondent is male.
32. Given that the person is pretty happy, the respondent is female.
33. Given that the person is not too happy, the respondent is female.
34. Given that the person is very happy, the respondent is female.
35. Given that the person is pretty happy, the respondent is male.
36. Given that the person is not too happy, the respondent is male.

Checkpoint Answers

1. $1/13 \approx .077$
2. About .7321
3. (a) About .1375 (b) About .4007
4. (a) About .3404 (b) About .1368

CHAPTER 8 Summary and Review

Key Terms and Symbols

1. set braces	8.1 set	8.2 addition rule for counting	8.4 addition rule for probability
2. is an element of	element (member)	8.3 experiment	complement rule
3. is not an element of	empty set	trial	odds
4. empty set	set-builder notation	outcome	8.5 conditional probability
5. is a subset of	universal set	sample space	product rule of probability
6. is not a subset of	subset	event	probability tree
7. is a proper subset of	set operations	certain event	independent events
8. complement of set A	tree diagram	impossible event	dependent events
9. set intersection	Venn diagram	disjoint events	8.6 Bayes' formula
10. set union	complement	basic probability principle	
11. $P(E)$ probability of event E	intersection	relative frequency	
12. $P(F E)$ probability of event F , given that event E has occurred	disjoint sets	probability	
	union	probability distribution	